Financing the Government with Taxes or Inflation*

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Abstract

An increase in government expenditures generates debt that in the long run has to be financed with taxes or inflation. We show that the predictions of an increase in government expenditures change when the reaction of agents toward their demand for money is taken into account. In the model, agents change their demand for money by changing the frequency of exchanges of bonds and money. In standard cash-in-advance models, this frequency is fixed, usually at a quarter. With fixed frequency, financing expenditures with taxes or inflation implies similar effects on output, hours of work, and welfare. With endogenous frequency, financing expenditures with taxes and inflation produces opposing effects. Hours of work and output increase when expenditures are financed with inflation. Although output increases, the welfare cost is 2 percentage points higher when the increase in expenditures is financed with inflation.

JEL Codes: E30, E40, E50.

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1. Introduction

We show that the effects of an increase in government expenditures change when the reaction of agents toward the demand for money is taken into account. To study the effect of the demand for money, we let agents decide the time interval in which they exchange bonds and money. In a standard cash-in-advance model, this time interval is fixed, such as one quarter. Here, the time interval changes, which generates an elastic demand for money with respect to the interest rate and a better match to the data on interest rates and money. The only difference between a standard cash-in-advance model with capital and labor and the model considered here is that here the interval between trades change and the demand for money is elastic. As the demand for money changes with inflation, our results differ from previous estimates when changes in government policy imply changes in inflation.

Consider an increase in government expenditures from 10 percent of GDP to 11 percent of GDP financed with taxes or with inflation. Suppose first that the increase in government expenditures is financed through an increase in taxes (labor taxes in the model). As an increase in taxes does not increase inflation, considering fixed intervals between trades as in the standard cash-in-advance model or endogenous intervals as considered here implies small changes in predictions. Output and hours of work decrease slightly, about 0.3 percent, and the increase in government expenditures implies a welfare cost of 2.6 percent in terms on income.

Taking into account the changes in the demand for money implies different predictions when the increase in expenditures is financed with inflation. That is, with the seigniorage revenue raised from an increase in inflation. In this case, output and hours of work increase 1.5 percent and the welfare cost increases to 3.7 percent in terms of income. With fixed periods, output and hours of work are approximately constant and the welfare cost is 2.5 percent. In terms of inflation, considering fixed intervals implies an increase in inflation to 6.6 percentage points per year whereas endogenous periods imply an increase in inflation to 12 percentage points per year. The difference of the change in inflation comes from the fact that the demand for money decreases with inflation, which forces the inflation rate to increase to imply the same level of seigniorage. The decrease in the demand for money is better captured with endogenous periods between exchanges of bonds or money.

Although output increases with inflation, the welfare loss is higher when the increase in government expenditures is financed with inflation. The welfare cost of financing the same increase in expenditures with inflation instead of taxes is equal to 1.8 percent in terms of income. That is, an agent has to be compensated with 1.8 percent of income to live in an economy in which the government finances the same increase in government expenditures with inflation instead of with taxes. 1.8 percent of income is equivalent to more 260 billion dollars every year or to more than two thousand dollars distributed to every household in the United States every year indefinitely (2010 dollars, data from the BEA and from the US Census Bureau).

The reason for the difference in results with fixed or endogenous trading intervals is difference in the use of resources. The agents in the model have to pay a cost to transform bonds into money, as in Baumol (1952) and Tobin (1956). We interpret this cost as a cost to obtain financial services. The only friction in the model is the cost to obtain financial services. In particular, prices and wages are flexible.

In order to decrease the demand for money, the agents divert resources to financial services. An economy with higher inflation may have higher output because more of its output is used to cover financial services. There is a government expenditures multiplier slightly larger than one.[1]

Output increases but the increase is used to increase financial services. An im-

[1] Ramey (2011) concludes that the available evidence to a government expenditures multiplier between 0.8 and 1.5. See also the discussion in Hall (2009) and Woodford (2010).

portant service for because it allows agents to decrease the real demand for money. But it is a service that does not increase welfare, as welfare is derived from consumption goods. Although output increases, the predicted welfare losses are larger with endogenous periods and inflation.

In addition to generating a distortion between consumption and leisure, inflation generates a distortion on the decision toward the demand for money. This distortion is concealed in models with fixed trading period, as agents cannot increase consumption of financial services.

In standard cash-in-advance models, agents sell interest-bearing bonds for money in every period to cover goods purchases for the following period. This is the case, for example, in Cooley and Hansen (1989, 1991). However, the fact that agents make bond sales and use all money proceeds in one period implies small variation in velocity, which is contrary to what we observe in the data. Even with cash and credit goods and with different assumption about the expectations of shocks, velocity varies little, as shown in Hodrick, Kocherlakota, and Lucas (1991).

A solution for the small variation in velocity is to allow agents to use the money from bond sales only after a larger time interval. This is the case of the models in Grossman and Weiss (1983) and Rotemberg (1984), in which agents are able to use the money from bond sales only with a two-period interval. Alvarez, Atkeson, and Edmond (2009) increase the size of the time interval and show that a model with these characteristics is able to fit the short-run variation in velocity.

Having fixed intervals between bond sales, however, still implies small variation in velocity in the long run. With fixed intervals, an increase in inflation implies changes in the labor supply and in the use of credit and cash goods, but the real demand for money remains approximately unresponsive in the long run. Silva (2012) shows that allowing agents to change the interval between bond sales for money implies an elastic demand for money and a better fit to the long run data.

Our objective is to obtain predictions for the long run effects of financing an increase in government expenditures in different ways. We compare the predictions of the model with fixed and with endogenous time intervals between trades. We find that the model with endogenous time intervals implies different predictions for the effects of financing the debt with taxes and inflation.

An analyst considering fixed intervals would conclude that financing an increase in government expenditures with taxes or inflation generate similar effects. Taking into account that agents change their demand for money, a fact observed in the data, leads to different predictions for the effects of an increase in government expenditures. Especially, it leads to higher predictions of welfare losses when the increase in expenditures is financed with inflation.

2. The Model

We extend the general equilibrium Baumol-Tobin model in Silva (2012). Money must be used to purchase goods, only bonds receive interest payments, and there is a cost to transfer money from bond sales to the goods market. Agents accumulate bonds during a certain time and exchange bonds for money infrequently. The infrequent sales of bonds for money occur as in the models of Grossman and Weiss (1983), Rotemberg (1984) and Alvarez, Atkeson, and Edmond (2009). The difference from these models is that the timing of the financial transfers is endogenous. Also, the model has capital and labor and a government that finances government expenditures with labor income taxes and seigniorage. The model can also be understood as a cash-in-advance model with capital and labor with the additional decision on the size of the holding periods. Apart from the heterogeneity of agents and the decision on the holding periods, the model is similar to the models in Cooley and Hansen (1989) and Cooley (1995).

Time is continuous and denoted by $t \in [0, \infty)$. At any moment there are markets for assets, for the consumption good, and for labor. There are three assets, money, claims to physical capital, and nominal bonds. The markets for assets and the market for the good are physically separated.

There is an unit mass of infinitely lived agents and with preferences over consumption and leisure. Agents have two financial accounts: a brokerage account and a bank account. They hold assets in the brokerage account and money in the bank account. We assume that readjustments in the brokerage account have a fixed cost. As only money can be used to buy goods, agents need to maintain an inventory of money in their bank account large enough to pay for their flow of consumption expenditures until the next transfer of funds.

Let M_0 denote money in the bank account at time zero. Let B_0 denote nominal bonds and k_0 claims to physical capital, both in the brokerage account at time zero. Index agents by $s = (M_0, B_0, k_0)$. The agents pay a cost Γ in goods to transfer resources between the brokerage account and the bank account. Γ represents a fixed cost of portfolio adjustment. Let $T_j(s)$, $j = 1, 2, ..$, denote the times of the transfers of agent s. Let $P(t)$ denote the price level. At $T_j(s)$, agent s pays $P(T_j(s))\Gamma$ to make a transfer between the brokerage account and the bank account. The agents choose the times $T_j(s)$ of the transfers.

The consumption good is produced by firms. Firms are perfect competitors. They hire labor and rent capital to produce the good. The production function is given by $Y(t) = Y_0 K(t)^{\theta} H(t)^{1-\theta}$, where $0 < \theta < 1$ and $K(t)$ and $H(t)$ are the aggregate quantities of capital and hours of work at time t. Capital depreciates at the rate δ, $0 < \delta < 1$. Let the transfer cost be given by $\Gamma = \gamma Y(t)$, linear in income. With this, the budget constraint of the agents and the demand for money will be linear in income. The income elasticity of the demand for money will be equal to one, which matches the evidence as stated in Lucas (2000) and others.

The agent is a composition of a shopper, a trader, and a worker, as in Lucas (1990). The shopper uses money in the bank account to buy goods, the trader manages the brokerage account, and the worker supplies labor to the firms. The firms transfer their sales proceeds to their brokerage accounts and convert them into bonds.[2]

The firms pay $w(t)\,h(t,s)$ and $r^k(t)\,k(t,s)$ to the worker for the hours of work $h(t,s)$ and capital $k(t,s)$ supplied, $w(t)$ are real wages and $r^k(t)$ is the real interest rate on capital. The firms make the payments with a transfer from the brokerage account of the firm to the brokerage account of the agent. At the time of the transfer, the government collects $\tau_L w(t)\,h(t,s)$ in labor income taxes. With the payments of the firm, the brokerage account of the worker is credited by $(1-\tau_L)\,w(t)\,h(t,s)+r^k(t)\,k(t,s)$. These credits can be used at the same date for purchases of bonds.

The government offers bonds that pay a nominal interest rate $r(t)$. Let the price of a bond at time zero be given by $Q(t)$, with $Q(0)=1$. The nominal interest rate is $r(t)\equiv -d\log Q(t)/dt$. Let inflation be denoted by $\pi(t)$, $\pi(t)=d\log P(t)/dt$. To avoid the opportunity of arbitrage between bonds and capital, the nominal interest rate and the payment of claims to capital satisfies $r(t)-\pi(t)=r^k(t)-\delta$. That is, the rate of return on bonds must be equal to the real return on physical capital discounted by depreciation. With this condition satisfied, the agents are indifferent between converting their income into bonds or capital.

Money holdings at time t of agent s are denoted by $M(t,s)$. Money holdings just after a transfer are denoted by $M^+(T_j(s),s)$ and they are equal to $\lim_{t\to T_j,t>T_j} M(t,s)$. Analogously, $M^-(T_j(s),s)=\lim_{t\to T_j,t<T_j} M(t,s)$ denotes money just before a transfer. The net transfer from the brokerage account to the bank account is given by M^+-M^-. If $M^+<M^-$, the agent makes a negative net transfer, a transfer from the bank account to the brokerage account, immediately converted into bonds. Money holdings in the brokerage account are zero, as bonds receive interest and it is not possible to buy goods directly with money in the brokerage account. All money holdings

[2] In Silva (2012), the firms keep a fraction a of the sales proceeds in money and transfer the remaining fraction $1-a$ to their brokerage accounts of the workers, $0\leq a<1$. However, the value of a has little impact on the welfare cost, on the demand for money and on other equilibrium values.

are in the bank account. To have M^{+} just after a transfer at $T_j(s)$, agent s needs to transfer $M^{+}-M^{-}+P(T_j(s))\Gamma$ to the bank account, $P(T_j(s))\Gamma$ is used to buy goods to pay the transfer cost.

Define a holding period as the interval between two consecutive transfer times, that is $[T_j(s), T_{j+1}(s))$. The first time agent s adjusts its portfolio of bonds is $T_1(s)$ and the first holding period of agent s is $[0, T_1(s))$. To simplify the exposition, let $T_0(s) \equiv 0$, but there is not a transfer at $t=0$, unless $T_1(s)=0$.

Denote $B^{-}(T_j(s), s)$, $B^{+}(T_j(s), s)$, $k^{-}(T_j(s), s)$, and $k^{+}(T_j(s), s)$ the quantities of bonds and capital just before and just after a transfer. These variables are defined in a similar way as defined for money. During a holding period, bond holdings and capital holdings of agent s follow

$$\begin{aligned} \dot{B}(t,s) &= r(t)B(t)+P(t)(1-\tau_L)w(t), \\ \dot{k}(t,s) &= \left(r^k(t)-\delta\right)k(t,s). \end{aligned} \tag{1}$$

Equation (1) states the labor income is converted into nominal bonds. However, the agent is indifferent if part of the labor income is converted into capital, as $r(t)-\pi(t)=r^k(t)-\delta$.

At each date $T_j(s)$, $j=1,2,...$, agent s readjusts its portfolio. At the time of a transfer $T_j(s)$, the quantities of money, bonds, and capital satisfy

$$M^{+}(T_j)+B^{+}(T_j)+P(T_j)k^{+}(T_j)+P(T_j)\Gamma = M^{-}(T_j)+B^{-}(T_j)+P(T_j)k^{-}(T_j) \tag{2}$$

, $j=1,2,...$ The portfolio chosen plus the real cost of readjusting must be equal to the current wealth. With the evolution of bonds and capital given by (1), we can write $B^{-}(T_j)$ and $k^{-}(T_j)$ as a function of the interest payments accrued during a holding period $[T_{j-1}, T_j)$. Substituting recursively and using the no-Ponzi conditions

$\lim_{j\to+\infty} Q(T_j) B^+(T_j) = 0$ and $\lim_{j\to+\infty} Q(T_j) P(T_j) k^+(T_j) = 0$, we obtain the present value constraint

$$\sum_{j=1}^{\infty} Q(T_j(s)) \left[M^+(T_j(s), s) + P(T_j)\Gamma\right] \leq \sum_{j=1}^{\infty} Q(T_j(s)) M^-(T_j) + W_0(s), \quad (3)$$

where $W_0(s) = B_0 + P_0 k_0 + \int_0^\infty Q(t) P(t)(1-\tau_L) w(t) h(t,s)\, dt$. The constraint (3) states that the present value of money transfers and transfer fees is equal to the present value of deposits in the brokerage account, including initial bond and capital holdings.

In addition to the present value budget constraint (3), the agents face a cash-in-advance constraint

$$\dot{M}(t,s) = -P(t) c(t,s), t \geq 0, t \neq T_1(s), T_2(s), ... \quad (4)$$

This constraint shows the transactions role of money: agents need money to buy goods. At $t = T_1(s), T_2(s), ...$, constraint (4) is replaced by $\dot{M}(T_j(s), s)^+ = -P(T_j(s)) c^+(T_j(s))$, where $\dot{M}(T_j(s), s)^+$ is the right derivative of $M(t,s)$ with respect to time at $t = T_j(s)$ and $c^+(T_j(s))$ is consumption just after the transfer.

The agents choose consumption $c(t,s)$, hours of work $h(t,s)$, money in the bank account $M(t,s)$, and the transfer times $T_j(s)$, $j = 1, 2, ...$ They make this decision at time zero given the paths of the interest rate and of the price level. The maximization problem of agent $s = (M_0, k_0, B_0)$ is then given by

$$\max_{c,h,T_j,M} \sum_{j=0}^{\infty} \int_{T_j(s)}^{T_{j+1}(s)} e^{-\rho t} u(c(t,s), h(t,s))\, dt \quad (5)$$

subject to (3), (4), $M(t,s) \geq 0$, and $T_{j+1}(s) \geq T_j(s)$, given $M_0 \geq 0$. The parameter $\rho > 0$ is the intertemporal rate of discount. The utility function is $u(c(t,s)) =$

$\log c(t,s) + \alpha \log(1 - h(t,s))$. Preferences are a function of goods and hour of work only, the transfer cost does not enter the utility function. These preferences are derived from the King, Plosser and Rebelo (1988) preferences $u(c,h) = \frac{[c(1-h)^{\alpha}]^{1-1/\eta}}{1-1/\eta}$, with $\eta \to 1$, which are compatible with a balanced growth path.[3]

As bonds receive interest and money does not, the agents transfer the exact amount of money to consume until the next transfer. That is, the agents adjust $M^{+}(T_j)$, T_j, and T_{j+1} to obtain $M^{-}(T_{j+1}) = 0$, $j \geq 1$. We can still have $M^{-}(T_1) > 0$ as M_0 is given rather than being a choice. Using (4) and $M^{-}(T_{j+1}) = 0$ for $j \geq 1$, money just after the transfer at T_j is

$$M^{+}(T_j(s), s) = \int_{T_j}^{T_{j+1}} P(t)\, c(t,s)\, dt,\ j = 1, 2, ... \tag{6}$$

The government makes consumption expenditures G, taxes labor income at the rate τ_L, and issues nominal bonds $B^G(t)$ and money $M(t)$. The government controls the aggregate money supply at each time t by making exchanges of bonds and money in the asset markets. The financial responsibilities of the government at time t satisfy the period budget constraint

$$r(t)\, B(t) + P(t)\, G = \dot{B}(t) + \tau_L P(t)\, w(t)\, H(t) + \dot{M}(t)\,. \tag{7}$$

That is, the government finances its responsibilities $r(t)\, B(t) + P(t)\, G$ by issuing new bonds, using the revenues from labor taxes, and by issuing money. With the condition $\lim_{t\to\infty} B(t)\, e^{-rt} = 0$, government budget constraint in present value is given by

$$B_0^G + \int_0^{\infty} Q(t)\, P(t)\, G dt = \int_0^{\infty} Q(t)\, \tau_L w(t)\, H(t)\, dt + \int_0^{\infty} Q(t)\, P(t)\, \frac{\dot{M}(t)}{P(t)} dt. \tag{8}$$

[3]We also have a version of the model with $\eta \neq 1$ and with the Greenwood, Hercowitz, and Huffman (1988) preferences. Silva (2012) has a version of the model with indivisible labor, analyzed by Hansen (1985).

Seigniorage is equal to the real resources obtained by issuing money, given by $\frac{\dot{M}(t)}{P(t)}$.

The market clearing conditions for money and bonds are $M(t) = \int M(t,s)\,dF(s)$ and $B_0^G = \int B_0(s)\,dF(s)$, where F is a given distribution of s. The market clearing condition for goods takes into account the goods used to pay the transfer cost. Let $A(t,\delta) \equiv \{s : T_j(s) \in [t, t+\delta]\}$ represent the set of agents that make a transfer during $[t, t+\delta]$. The number of goods used on average during $[t, t+\delta]$ to pay the transfer cost is then given by $\int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s)$. Taking the limit to obtain the number of goods used at time t yields that the market clearing condition for goods is given by $\int c(t,s)\,dF(s) + \dot{K}(t) + \delta K(t) + G + \lim_{\delta\to 0} \int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s) = Y$. The market clearing for capital and hours of work are $K(t) = \int k(t,s)\,dF(s)$ and $H(t) = \int h(t,s)\,dF(s)$.

An equilibrium is defined as prices $P(t)$, $Q(t)$, allocations $c(t,s)$, $M(t,s)$, $B(t,s)$, $k(t,s)$, transfer times $T_j(s)$, $j = 1, 2, \ldots$, and a distribution of agents F such that (i) $c(t,s)$, $M(t,s)$, $B(t,s)$, $k(t,s)$, and $T_j(s)$ solve the maximization problem (5) given $P(t)$, $r(t)$, and $r^k(t)$ for all $t \geq 0$ and s in the support of F; (ii) the government budget constraint holds; and (iii) the market clearing conditions for money, bonds, goods, capital, and hours of work hold.

3. Financing the Government

We study the long run effects of financing the government with taxes or inflation. Focus, therefore, on an equilibrium in the steady state, an equilibrium in which the nominal interest rate is constant at r and the inflation rate is constant at π. Moreover, the aggregate quantities of capital and labor are constant at K and H, and output is constant.

The transfer cost Γ and the payment of interest on capital and bonds make agents follow (S,s) policies on consumption, money, capital, and bonds. For money, agent s makes a transfer at T_j to obtain money $M^+(T_j)$ at the beginning of a holding period. The agent then lets money holdings decrease until $M(t,s) = 0$, just before a new transfer at T_{j+1}. Symmetrically, the quantity of bonds $B^{+}(T_j)$ is relatively low and it increases at the rate r until it reaches $B^{-}(T_{j+1})$, just before T_{j+1}. The same applies to the behavior of $k(t,s)$. We assume that, in the steady state, the agents follow the same pattern of consumption along a holding period. With constant inflation and interest rates, it implies that the agents start a holding period with a certain value of consumption, $c^{+}(T_j)$, and that it decreases until the value $c^{-}(T_{j+1})$, just before the transfer at T_{j+1}. The agents look the same along holding periods, although they can be in different positions of the holding period.[4]

As the agents follow the same pattern of consumption along holding periods, they must also have the same interval between holding periods N. Let $n \in [0, N)$ denote the position of an agent along a holding period and reindex agents by n. Agent n makes the first transfer at $T_1(n) = n$, and then makes transfers at $n+N$, $n+2N$ and so on. Given that the agents have the same consumption profile across holding periods, the distribution of agents along $[0, N)$ compatible with a steady state equilibrium is a uniform distribution, with density $1/N$. We can then solve backwards to find the initial values of M_0, B_0, and k_0 for each agent $n \in [0, N)$ that implies that the economy is in the steady state since $t = 0$.[5]

To characterize the pattern of consumption of each agent, consider the first order conditions of the individual maximization problem (5) with respect to consumption. These first order conditions imply

$$c(t,n) = \frac{e^{-\rho t}}{P(t)\,\lambda(n)\,Q(T_j)},\ t \in (T_j, T_{j+1}),\ j = 1, 2, ..., \tag{9}$$

where $\lambda(n)$ is the Lagrange multiplier associated to the budget constraint (3). Let

[4] For a description of different applications of (S,s) models in economics, see Caplin and Leahy (2010).

[5] See Silva (2011, 2012) for an additional analysis on the distribution of agents in the steady state. Silva (2011) has the characterization of M_0 and B_0 for each agent n. The characterization of k_0 is obtained analogously.

c_0 denote consumption just after a transfer. In the steady state, $P(t) = P_0 e^{\pi t}$, for a given initial price level P_0, and $Q(T_j) = e^{-rT_j}$. Therefore, rewriting (9), individual consumption along holding periods is given by

$$c(t,n) = c_0 e^{(r-\pi-\rho)t} e^{-r(t-T_j)}, \tag{10}$$

taking the largest j such that $t \in [T_j(n), T_{j+1}(n)]$. We find aggregate consumption by integrating (10), using the fact that the distribution of agents along $[0, N)$ is uniform. Aggregate consumption is then

$$C(t) = c_0 e^{(r-\pi-\rho)t} \frac{1-e^{-rN}}{rN}. \tag{11}$$

As aggregate consumption is constant in the steady state, the nominal interest rate and the inflation rate that are compatible with the steady state are such that

$$r = \rho + \pi. \tag{12}$$

From (10) and (11), $c(t,n)$ decreases in the interval $t \in [T_j, T_{j+1})$ at the rate r. On the other hand, aggregate consumption is constant at $c_0 \dfrac{1-e^{-rN}}{rN}$. The individual behavior given by $c(t,n)$ is very different from the aggregate behavior, as in other (S,s) models. In particular, the variability of consumption is much larger at the individual level.

Given the production function $Y = Y_0 K^\theta H^{1-\theta}$, profit maximization implies that $w = (1-\theta) Y_0 (K/H)^\theta$ and $r^k = \theta Y_0 (K/H)^{-(1-\theta)}$, constant in the steady state. With the non-arbitrage condition $r^k - \delta = r - \pi$ and (12), we obtain that $K/H = [\theta Y_0/(\rho + \delta)]^{1/(1-\theta)}$. Therefore, $K/Y = \theta/(\rho+\delta)$. As K is constant in the steady state, the investment output ratio $(\dot{K} + \delta K)/Y$ is given by $\delta\theta/(\rho+\delta)$.

The first order conditions for $h(t,n)$ imply

$$1 - h(t,n) = \frac{\alpha c_0}{(1-\tau_L)\, w}. \tag{13}$$

Therefore, as wages are constant in the steady state, individual hours of work are constant along holding periods. As there is a unit mass of agents, $H = h$. With the expression of wages, we obtain the equilibrium value of the hours of work,

$$h = 1 - \frac{\alpha c_0}{(1-\tau_L)(1-\theta)\, Y_0 (K/H)^{\theta}}. \tag{14}$$

As c_0 depends on r and N, equation (14) determines hours of work as a function of r and N.

The market clearing condition for goods in the steady state is $C(t) + \delta K + G + \frac{1}{N}\gamma Y = Y$. This equation implies

$$c_0 \frac{1-e^{-rN}}{rN} + \delta \frac{K}{Y} Y + G + \frac{1}{N}\gamma Y = Y. \tag{15}$$

Dividing by Y, we obtain an expression for the consumption-income ratio $\hat{c}_0 \equiv c_0/Y$ in terms of N and the ratio between government expenditures and output,

$$\hat{c}_0 \frac{1-e^{-rN}}{rN} + \delta \frac{K}{Y} + v + \frac{1}{N}\gamma = 1, \tag{16}$$

where $v = G/Y$ and $K/Y = \theta/(\rho+\delta)$. Equation (16) implies that

$$\hat{c}_0 = \left(1 - \delta \frac{K}{Y} - v - \frac{1}{N}\gamma\right) \left(\frac{1-e^{-rN}}{rN}\right)^{-1}. \tag{17}$$

The holding period N is obtained with the first order conditions for $T_j(n)$. As derived in the appendix, the optimal holding period N must satisfy

$$c_0 r N \left(1 - \frac{1 - e^{-\rho N}}{\rho N}\right) = \rho \gamma Y. \tag{18}$$

Or, in terms of the consumption-income ratio $\hat{c}_0 \equiv c_0/Y$, $\hat{c}_0 r N \left(1 - \dfrac{1 - e^{-\rho N}}{\rho N}\right) = \rho\gamma$, where $\hat{c}_0$ is given by (17). This equation characterizes the value of N given the ratio $v = G/Y$. Following the argument in Silva (2011), $\partial N/\gamma > 0$ and $\partial N/\partial r < 0$. That is, the holding period increases with the transfer cost and decreases with the interest rate.

The aggregate demand for money is given by $M(t) = \dfrac{1}{N} \int M(t,n)\, dn$. Given individual consumption $c(t,n)$ for an agent that has made a transfer at T_j, individual money holdings at t are given by $M(t,n) = \int_t^{T_{j+1}} P(\tau)\, c(\tau,n)\, d\tau$, $\tau \in (T_j, T_{j+1})$, using the cash-in-advance constraint (4). At any time t there will be agents in their holding period $j+1$ and others in their holding period j. Taking this fact into account and the behavior of $c(t,n)$ in (10), it is possible to express the money-income ratio $m = M/(PY)$ in terms of r, N, and the consumption-income ratio $\hat{c}_0$. As derived in the appendix, the expression of the money-income ratio is

$$m(r) = \frac{\hat{c}_0(r,N)\, e^{-rN(r)}}{\rho} \left[\frac{e^{rN(r)} - 1}{rN(r)} - \frac{e^{(r-\rho)N(r)} - 1}{(r-\rho)\, N(r)}\right], \tag{19}$$

where $\hat{c}_0$ is given by (17). The values of m and N are written as $m(r)$ and $N(r)$ to emphasize their dependency on the nominal interest rate r. Real balances are then given by $\dfrac{M}{P} = m(r)\, Y$. As output Y is constant in the steady state, the growth rate of the money supply must be equal to the rate of inflation, π.

The values of equation (19) can be compared with the data on interest rates and money-income ratio. This is done in Figure 1. The data in the figure is similar to the data used in Lucas (2000), Lagos and Wright (2005), and Ireland (2009). In particular, we use commercial paper rate for the nominal interest rate and M1 for the monetary aggregate. We use the same data to facilitate the comparison of the results. Especially, to facilitate the comparison of the welfare cost values to be found in the next section. Equation (19) implies an interest-elasticity of the demand for money around $-1/2$ and semi-elasticity of -12.5. Lucas (2000), Guerron-Quintana, Alvarez and Lippi (2009) and others argue that the evidence on interest rates and money indicate a long-run interest-elasticity of $-1/2$.

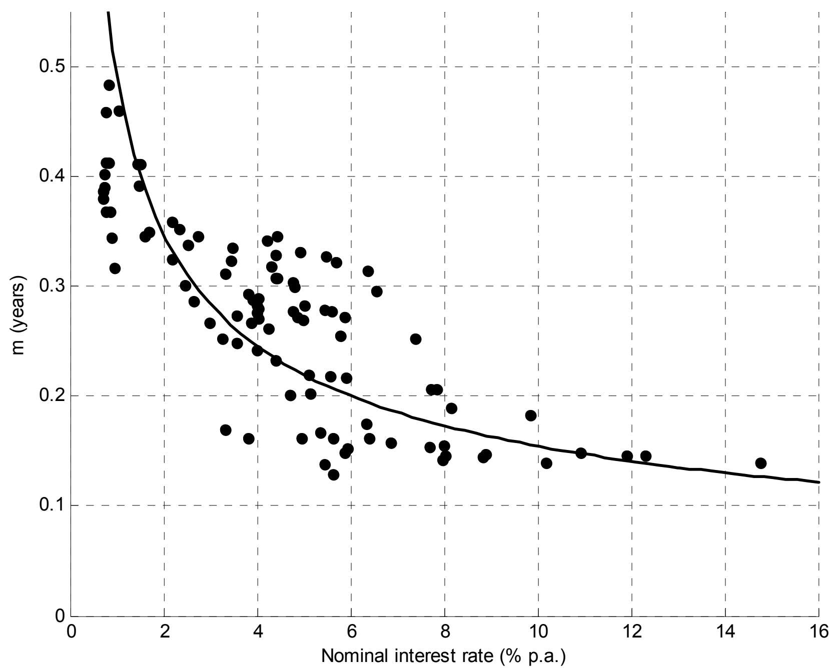

FIG. 1.

To close the model, we need an equation that links government expenditures with the labor tax τ_L. The time t government budget constraint in the steady state is given by $rB(t) + P(t)G = \dot{B}(t) + \tau_L P(t) wH + \dot{M}(t)$. In real terms, this equation implies $rb(t) + G = b(t)\frac{\dot{B}(t)}{B(t)} + \tau_L wH + \frac{M(t)}{P(t)}\frac{\dot{M}(t)}{M(t)}$, where $b(t) = \frac{B(t)}{P(t)}$. In the steady state, $\pi = \dot{P}/P = \dot{M}/M$ and $\dot{B}(t)/B(t) = r$. Therefore, we obtain that government expenditures must satisfy the constraint

$$G = \tau_L w H + \pi \frac{M}{P}, \tag{20}$$

where $\pi = r - \rho$. Equation (20) states that government expenditures must be financed through revenues from labor income taxes $\tau_L w H$ or through seigniorage $\pi \frac{M}{P}$.

Equation (20) completes the characterization of the equilibrium. Given a value G for government expenditures, we have a system of equations to obtain the equilibrium variables N, h, c_0, r, τ_L, m, and $\dfrac{M}{P}$.

4. An Increase in Government Expenditures

Suppose that the economy is in a long run equilibrium, following the equations for the steady state as described in section 3 for a given value of government expenditures. We now calculate the effects of an increase in government expenditures ΔG, an increase in expenditures on goods and services by the government. Given the equilibrium for an initial labor tax τ_L and an initial interest rate r, we change the value of G and recalculate the values of τ_L and r so that the system of equations is satisfied for the new value of G.

We change τ_L and r separately. That is, for financing ΔG with labor income taxes, we maintain the value of r at its initial value and change τ_L such that the government budget constraint (20) and the remaining equations for the equilibrium are again satisfied. Analogously, for financing ΔG with inflation, we maintain τ_L and find the new interest rate r such that (20) and the remaining equations for the equilibrium are satisfied. The new inflation rate is given by $r - \rho$.

Welfare Cost

The welfare cost of a fiscal policy G_A with respect to the policy with expenditures

G_B is defined as the income compensation $w(G_A)$ to leave agents indifferent between an economy with G_A and an economy with G_B. The value of $w(G_A)$ is such that $U^T[c(G_A,(1+w(G_A))Y(G_A)), h(G_A)] = U^T[c(G_B, Y(G_B)), h(G_B)]$, where U^T is the aggregate utility for all agents with equal weight. The preferences $u(c,h) = \log c + \alpha \log(1-h)$ imply

$$1+w(G_A) = \frac{c_0(G_B)}{c_0(G_A)}\left(\frac{1-h(G_B)}{1-h(G_A)}\right)^{\alpha}\exp\left(\frac{r(G_A)N(G_A)}{2} - \frac{r(G_B)N(G_B)}{2}\right), \quad (21)$$

where $c_0(G_i)$, $h(G_i)$, $N(G_i)$, and $r(G_i)$, $i = A, B$, are given by the equilibrium conditions given in section 3. A fiscal policy A or B may identify a value for government expenditures G and method of financing of ΔG. Government expenditures do not enter the utility. Therefore, an increase in G always imply a positive welfare cost with respect to the economy with lower G. However, by fixing a value for G, we can compare an economy in which A denotes financing with inflation and an economy in which B denotes financing with labor income taxes.

The experiment that we do is to increase G such that the ratio of government expenditures to output increases from 10 percent to about 11 percent. This is done by first calibrating the economy for $G/Y = 10$ percent and then increasing G by 15 percent, $G' = G \times 1.15$. We cannot fix G' and at the same time fix the beginning and end values of G/Y, as the behavior of output depends on the change in policy. The increase in G of 15 percent implies that G/Y increases to 11.32 percent for the case with N endogenous and financing with inflation.

Calibration

We set standard values for the parameters. As in Cooley and Hansen (1989), we set $\theta = 0.36$ for the parameter for capital and $\delta = 0.10$ for the depreciation. We set $\alpha = 1.75$, such that hours of work are equal to 0.3 when $G/Y = 10$ percent and the interest rate is equal to the geometric mean of the nominal interest rate in the data,

$r_{Avg} = 3.64$ percent p.a. As in Lucas (2000), we set ρ such that an interest rate of 3 percent p.a. implies zero inflation, that is, $\rho = 3$ percent p.a.

The parameters ρ and the value of G/Y affect the demand for money m, but the main parameter that affects the demand for money is the transfer cost γ. A higher value of γ shifts upward the demand for money (it does not affect the interest-rate elasticity of the demand for money in an important way). We set γ so that the demand for money in (19) passes through the geometric mean of the data, $r_{Avg} = 3.64$ percent p.a. and $m_{Avg} = 0.2573$ year (that is, the average money-income ratio in the data implies that the average agent in the U.S. holds the income of about one quarter in money or the average velocity is equal to $1/0.25 = 4$ per year). Similarly, Lucas (2000) and Silva (2012) determine the parameters for the demand for money such that the theoretical demand for money passes through the geometric average of the data. Also, Alvarez et al. (2009) obtains the holding period (exogenous in their case) such that the theoretical demand for money approximates the average velocity in the data. This method implies $\gamma = 2.49$. As argued in Silva (2012), this value of γ implies that, for $r = 4$ percent p.a., an agent in the U.S. devotes on average about 22 minutes per week to financial services.

As in other models with market segmentation, the value of N implied by the parameters is large. For $r = 5$ percent p.a. (inflation rate of 2 percent per year), the value of N is equal to 261 days. For a comparison, Alvarez et al. (2009) use values of N of 24 months and 36 months (they use M2 instead of M1, which requires a higher value for N). Notice that N is the interval between exchanges of high-yielding assets to low-yielding assets, it is not the interval between ATM withdrawals. Christiano et al. (1996), Vissing-Jorgensen (2002), and Alvarez et al. (2009) show evidence that, in fact, firms and households rebalance their portfolios infrequently, in a way that explains the values found for N.

Edmond and Weill (2008) argue that the large values for holding periods in market segmentation models are an effect of the aggregation assumptions in these models. Agents in the model encompass firms and households, and firms hold a large portion of money in the economy (Bover and Watson 2005). Moreover, the use of cash by firms has increased across firms (Bates et al. 2009). Also, the parameters reflect the large money holdings that are found in the data (according to the data, $m = 0.25$, which imply about 10 thousand dollars per person in money in the U.S., or about 30 thousand dollars per household).

Given $G/Y = 0.10$, $r = 3.64$ percent p.a., and the other parameters of the model, the labor income tax before the increase in government expenditures is given by $\tau_L = 15.37$ percent. Inflation before the increase in G is given by $r_{Avg} - \rho = 0.64$ percent p.a. The government budget constraint $G = \tau_L wH + \pi \frac{M}{P}$ implies $G/Y = \tau_L (1-\theta) + \pi m$. The parameters imply that the ratio of taxes over GDP is $\tau_L (1-\theta) = 9.835\%$ and ratio of seigniorage over GDP is given by $\pi m = 0.165\%$. The sum of the the two sources of government revenues implies $G/Y = 10$ percent.

An Increase in Government Expenditures

We now set the economy with the initial situation, such that $G/Y = 10$ percent and increase the value of G by 15%. That is, $G' = 1.15G$. This increase in G increases the ratio G/Y about 1 percentage point, from 10 percent to 11 percent.

We study two ways of financing the increase in G: by increasing labor income taxes τ_L and by increasing the inflation rate π. In both cases, the government budget constraint $G = \tau_L wH + \pi \frac{M}{P}$ must be satisfied for the new value of G. Moreover, we study the effects of the change in G in two cases. One in the case of fixed N and the other in the case of endogenous N. The case of fixed N approximates the analysis from a standard cash-in-advance model with fixed periods. It is not exactly as a standard cash-in-advance model because the agents can still smooth consumption during the interval N. So, the demand for money varies a little even with N fixed. However, the effects in this model and a model in which consumption cannot change are similar. What strongly changes the predictions of the model is the case in which N is endogenous. In this case, the demand for money decreases more strongly when inflation increases, a pattern compatible with the data. The results are in Table 1.

Table 1. Effects of a 15% increase in government expenditures

	***N* Fixed**	***N* Endogenous**
Financed with Taxes		
Output	-0.24%	-0.27%
Capital	-0.23%	-0.27%
Consumption	-2.73%	-2.74%
Hours of Work	-0.23%	-0.27%
Holding Period (days)	-	1.24%
Money-Income Ratio	-2.53%	-1.28%
Government Multiplier	-0.17	-0.18
Taxes after the change	17.76%	17.76%
Welfare Cost	2.59%	2.58%
Financed with Inflation		
Output	0.00%	1.50%
Capital	0.00%	1.53%
Consumption	-2.4%	-2.2%
Hours of Work	0.00%	1.53%
Holding Period (days)	-	-50%
Money-Income Ratio	-1.67%	-52%
Government Multiplier	0.00	1.04
Inflation after the change	6.6%	12.0%
Welfare Cost	2.53%	3.68%
Welfare Cost of financing with inflation instead of taxes	-0.06%	1.78%

Government expenditures increase from G to G*1.15. Welfare cost as an income compensation over the economy with high G. Inflation before the change: 0.64% p.a. Taxes before the change: 15.37%. N Fixed: optimal choice of N under r=3.64% p.a., the geometric mean of r in the period.

As shown in Table 1, considering only fixed period underestimates the inflation required to finance the increase in government expenditures. Moreover, considering fixed periods underestimates the welfare cost of financing the increase in government expenditures financed with inflation. Financing the same increase in government expenditures with inflation instead of labor taxes implies a welfare cost of about 1.8 percent in terms of income. Agents living in an economy that finances with inflation an increase from G/Y from 10 percent to 11 percent have to be compensated with 1.8 of their income every year to have the same welfare as agents that financed the same increase in G with labor taxes. The underestimation of the effects is such that a model with fixed N implies that there is a gain in financing the increase in G with inflation. The result is reversed with endogenous N.

The table also shows that supposing N fixed or endogenous matters little when the change in policy does not involve changes in inflation. The predictions are equivalent for N fixed or endogenous when ΔG is financed with taxes. Taxes increase from 15.37 percent to 17.76 percent in this case, but inflation is maintained constant at 0.64 percent p.a.

The predictions are different when the increase in government expenditures is financed with inflation. In this case, the opportunity cost of holding money increases and, therefore, the agents spend resources to decrease their real demand for money. With fixed periods, the agents can decrease slightly the demand for money by making consumption within holding periods steeper. This behavior also shows up with N endogenous, but most of the decrease in the demand for money is obtained by decreasing N. The value of N decreases 50% with the change in inflation. With N fixed, the money-income ratio decreases about 2 percent. With N endogenous, the money-income ratio decreases about 50%. An inflation of 12 percent p.a. would then make the money-income ratio decrease to the values in the 1980s, corresponding to the data points on the Southeast of Figure 1.

A surprising effect of considering N endogenous is the response of output after the increase in G. As shown in the table, output increase 1.5 percent after the increase in government expenditures, when the increase in government expenditures is financed with inflation. The response of output implies a government expenditures multiplier slightly above 1. On the other hand, an increase in labor taxes encourages agents to substitute away from labor toward leisure, which decreases output. This effect is found in Table 1, which shows a decrease in output when τ_L increases. Inflation has similar effects when the effect on the demand for money is not taken into account. However, when N is endogenous, the increase in financial services to generate a decrease in the demand for money implies an increase in output.

A multiplier slightly above 1 is compatible with the values discussed in Hall (2009) and Ramey (2011). Woodford (2011) discussess how the multiplier can be above 1 in models with sticky prices or sticky wages. Here, the multiplier is above 1 with flexible prices. The only friction in the model are the financial frictions. Although output increases, the welfare cost of the economy with high inflation is large.

5. Conclusions

We take into account that agents react to different fiscal policies by changing the demand for money. The demand for money decreases when inflation increases. Agents need to divert resources to financial services in order to decrease the demand for money.

The agents change their demand for money by increasing the frequency of bond trades. In contrast, standard cash-in-advance models assume that the frequency of trades is fixed. Letting the frequency of trades vary implies a more elastic demand for money and a better fit to the data.

Taking into account the changes in the demand for money imply different predictions about the effects of an increase in government purchases and about the effects of different forms of financing an increase in government expenditures. The government consumption multiplier is larger when the timing of the transactions is endogenous and when public consumption is financed with seigniorage. Although output is larger when the same increase in government expenditures is financed with inflation, as compared to the case of financing with labor income taxes, the welfare losses are larger.

References

Alvarez, Fernando, Andrew Atkeson, and Chris Edmond (2009). "Sluggish Responses of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand." *Quarterly Journal of Economics*, 124(3): 911-967.

Alvarez, Fernando, and Francesco Lippi (2009). "Financial Innovation and the Transactions Demand for Cash." *Econometrica*, 77(2): 363-402.

Bates, Thomas W., Kathleen M. Kahle, and Rene M. Stulz (2009). "Why Do U.S. Firms Hold so Much More Cash than They Used to?" *Journal of Finance*, 64(5): 1985-2021.

Baumol, William J. (1952). "The Transactions Demand for Cash: An Inventory Theoretic Approach." *Quarterly Journal of Economics*, 66(4): 545-556.

Bover, Olympia, and Nadine Watson (2005). "Are There Economies of Scale in the Demand for Money by Firms? Some Panel Data Estimates." *Journal of Monetary Economics*, 52(8): 1569-1589.

Caplin, Andrew, and John Leahy (2010). "Economic Theory and the World of Practice: A Celebration of the (S, s) Model." *Journal of Economic Perspectives*, 24(1): 183-202.

Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans (1996). "The Effects of Monetary Policy Shocks: Evidence from the Flow of Funds." *Review of Economics and Statistics*, 78(1): 16-34.

Cooley, Thomas F. (ed.) (1995). *Frontiers of Business Cycle Research.* Princeton: Princeton University Press.

Cooley, Thomas F., and Gary D. Hansen (1989). "The Inflation Tax in a Real Business Cycle Model." *American Economic Review*, 79(4): 733-748.

Cooley, Thomas F., and Gary D. Hansen (1991). "The Welfare Costs of Moderate Inflations." *Journal of Money, Credit and Banking*, 23(3): 483-503.

Edmond, Chris, and Pierre-Olivier Weill (2008). "Models of the Liquidity Effect." In *The New Palgrave Dictionary of Economics*, 2nd Edition, ed. S. N. Durlauf, and L. E. Blume. Palgrave Macmillan.

Friedman, Milton (1969). "The Optimum Quantity of Money." In Friedman, Milton, *The Optimum Quantity of Money and Other Essays.* Aldine: Chicago.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman (1988). "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review*, 78(3): 402-417.

Grossman, Sanford J., and Laurence Weiss (1983). "A Transactions-Based Model of the Monetary Transmission Mechanism." *American Economic Review*, 73(5): 871-880.

Guerron-Quintana, Pablo A. (2009). "Money Demand Heterogeneity and the Great Moderation." *Journal of Monetary Economics*, 56(2): 255-266.

Hall, Robert E. (2009). "By How Much Does GDP Rise If the Government Buys More Output?" *Brookings Papers on Economic Activity*: 183-231.

Hansen, Gary D. (1985). "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*, 16(3): 309-327.

Hodrick, Robert J., Narayana Kocherlakota, and Deborah Lucas (1991). "The Variability of Velocity in Cash-in-Advance Models." *Journal of Political Economy*, 99(2): 358-384.

Ireland, Peter N. (2009). "On the Welfare Cost of Inflation and the Recent Behavior of Money Demand." *American Economic Review*, 99(3): 1040-1052.

King, Robert G., Charles I. Plosser, and Sergio T. Rebelo (1988). "Production, Growth, and Business Cycles: I. The Basic Neoclassical Model." *Journal of Monetary Economics*, 21(2-3): 195-232.

Lagos, Ricardo, and Randall Wright (2005). "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, 113(3): 463-484.

Lucas, Robert E., Jr. (1990). "Liquidity and Interest Rates." *Journal of Economic Theory*, 50: 237-264.

Lucas, Robert E., Jr. (2000). "Inflation and Welfare." *Econometrica*, 68(2): 247-274.

Ramey, Valerie A. (2011). "Can Government Purchases Stimulate the Economy?" *Journal of Economic Literature*, 49(3): 673-685.

Rotemberg, Julio J. (1984). "A Monetary Equilibrium Model with Transactions Costs." *Journal of Political Economy*, 92(1): 40-58.

Silva, Andre C. (2011). "Individual and Aggregate Money Demands." Nova School of Business and Economics Working Paper 557.

Silva, Andre C. (2012). "Rebalancing Frequency and the Welfare Cost of Inflation." *American Economic Journal: Macroeconomics*, 4(2): 153-183.

Tobin, James (1956). "The Interest-Elasticity of Transactions Demand for Cash." *Review of Economics and Statistics*, 38(3): 241-247.

Vissing-Jorgensen, Annette (2002). "Towards an Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures." National Bureau of Economic Research, Working Paper 8884.

Woodford, Michael (2011). "Simple Analytics of the Government Expenditure Multiplier" *American Economic Journal: Macroeconomics*, 3: 1-35.